

## VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD

## B.E. (CBCS) III-Semester Main Examinations, December-2017

Engineering Mathematics-III
(Common to Civil, CSE, ECE \& Mech.)
Time: $\mathbf{3}$ hours
Max. Marks: 70
Note: Answer ALL questions in Part-A and any FIVE from Part-B

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\text { Part-A }(10 \times 2=20 \text { Marks })
$$

1. Write the Dirichlet's conditions for existence of Fourier series of a function $f(x)$ in $(\alpha, \alpha+2 \pi)$.
2. Find the coefficient $b_{1}$ in the half-range Fourier sine series of $f(x)= \begin{cases}1, & 0<x<\frac{1}{2} \\ 0, & \frac{1}{2} \leq x<1\end{cases}$
3. Solve $p-q=z^{2}$.
4. Find the complete integral of the partial differential equation $(p x+q y-z)^{2}=p^{2}+q^{2}$
5. Find $\Delta(x+\cos x)$, if $h=\pi$.
6. Using Euler's method, find the approximate value of $y(0.2)$ for the initial value problem $y^{\prime}=x^{2}+y^{2}, y(0)=1$.
7. Derive normal equations for fitting a straight line by the method of least squares.
8. The equations of two regression lines are $2 x-3 y=0$ and $4 y-5 x-8=0$. Find the mean values $\bar{x}$ and $\bar{y}$.
9. A fair die is tossed. Let the random variable $X$ denote the twice the number appearing on the die. Find the probability distribution of $X$.
10. If $M_{X}(t)=\frac{2}{2-t}$ is the moment generating function of a random variable $X$, find the variance of $X$.

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\text { Part-B }(5 \times 10=50 \text { Marks })
$$

11. a) Obtain the Fourier series for $f(x)=\left\{\begin{array}{cc}-\pi, & -\pi<x<0 \\ x, & 0<x<\pi\end{array}\right.$. Hence deduce that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\cdots=\frac{\pi^{2}}{8}$.
b) Express $f(x)=x$ as a cosine series in $0<x<2$.
12. a) Find all possible second order partial differential equations by eliminating the arbitrary constants $a, b, c$ from $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$.
b) A tightly stretched string with fixed end points $x=0$ and $x=\pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $0.03 \sin x-0.04 \sin 3 x$, find the displacement at any point of the string at any time $t$.
13. a) The following table gives the velocity $v$ of a particle at time $t$. Find its acceleration at $t=2$.

| $\mathrm{t}:$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{v}:$ | 4 | 6 | 16 | 34 | 60 | 94 | 131 |

b) Using Newton's divided difference formula, find the missing value from the following table:

| $x:$ | 1 | 2 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y:$ | 14 | 15 | 5 | - | 9 |

14. Find the coefficient of correlation and the equations of the two lines of regression from the following data:

| $x$ | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

15. a) If a continuous random variable $X$ has the distribution function
$F(x)=\left\{\begin{array}{cc}0, & x \leq 1 \\ k(x-1)^{4}, & 1<x \leq 3 \\ 1, & x>3\end{array}\right.$, find the i) probability density function $f(x)$ ii) $k$
and iii) mean.
b) Two independent samples of sizes 8 and 7 respectively had the following values of the variable:

$$
\begin{array}{lcccccccc}
\text { Sample 1: } & 9 & 11 & 13 & 11 & 15 & 9 & 12 & 14 \\
\text { Sample 2: } & 10 & 12 & 10 & 14 & 9 & 8 & 10 &
\end{array}
$$

Is the difference between the means of samples significant? (Given $t_{0.05}(13)=2.16$ )
16. a) Expand $f(x)=|\cos x|$ in Fourier series for $-\pi<x<\pi$.
b) Find the general solution of $x\left(z^{2}-y^{2}\right) p+y\left(x^{2}-z^{2}\right) q=z\left(y^{2}-x^{2}\right)$.
17. Answer any two of the following:
a) Find the cubic polynomial which takes the following values using Newton's backward interpolation formula.

| $\mathrm{x}:$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x}):$ | 0 | 2 | 1 | 10 |

b) If $\theta$ is the acute angle between the two regression lines, show that
$\tan \theta=\frac{1-r^{2} \quad \sigma_{x} \sigma_{y}}{r} \sigma_{x}^{2}+\sigma_{y}^{2}$.
c) If $X$ is a normal variate with mean 30 and standard deviation 5 , find the probabilities that i) $26 \leq X \leq 40$ and ii) $X \geq 45$.
(Given $P(0<z<2)=0.4772, P(0<z<0.8)=0.2881, P(0<z<3)=0.4987)$

